

PERFORMANCE OF THE NEW WCOS TECHNIQUE FOR THE TITAN SPECT FORMULATION

Katherine Royston & Alireza Haghighat
Nuclear Engineering Program, Virginia Tech,
Arlington, Virginia, USA

OUTLINE

- Research Objectives
- Introduction to SPECT
- The TITAN Code SPECT Formulation
- Development of the WCOS Algorithm
- Results – Accuracy & Parallel Performance
- Conclusions and Future Work



RESEARCH OBJECTIVES

- Exploit the advantages of deterministic codes over Monte Carlo methods
 - Statistical uncertainty
 - Computation time

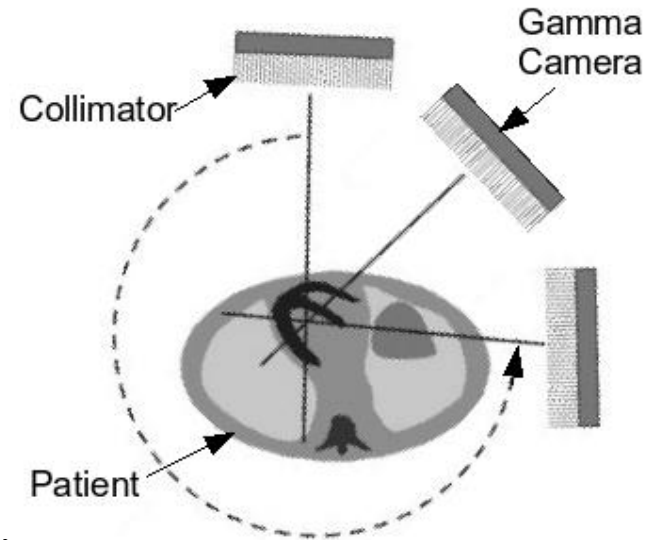
- This work specifically seeks to:
 - Benchmark the TITAN code's collimator representation
 - Better comprehend sensitivity to parameters
 - Improve upon the collimator representation's accuracy
 - Examine the parallel behavior of the code



INTRODUCTION TO SPECT

○ Single Photon Emission Computed Tomography

- 17 million procedures in the US in 2010
- Nuclear medicine imaging procedure used to examine myocardial perfusion, bone metabolism, thyroid function, etc.
- *Functional* imaging modality
- Radiopharmaceutical injected/ingested and localizes in a part of the body
- Emitted radiation detected at a gamma camera to form 2D projection images at different angles
- Collimator in front of the gamma camera provides spatial resolution
- Projection images can be reconstructed to form a 3D image of the radionuclide distribution



THE TITAN CODE

- Deterministic transport code* to solve the linear Boltzmann equation (LBE)

$$Hy = S + V$$

$$H = \hat{W} \cdot \nabla + S_t(\vec{r}, E) - \int_0^\infty dE' \int_{4\pi} d\Omega' S_s(\vec{r}, E' \rightarrow E, \hat{W}' \rightarrow \hat{W})$$

- Hybrid code allowing different solvers:
 - Discrete Ordinates (S_N) Method: discretize spatial domain into meshes (differencing scheme) and solve LBE in a discrete set of directions (quadrature set)
 - Characteristics Method (CM): discretize spatial domain into arbitrarily shaped regions and solve integral LBE along parallel directions (quadrature set)



THE TITAN SPECT FORMULATION

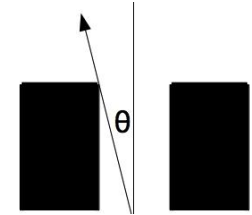
- Four-step hybrid S_N and simplified ray-tracing formulation:
 1. S_N transport calculation in the phantom with regular quadrature set
 2. Generation of fictitious quadrature set with circular ordinate splitting (COS) for a projection angle
 3. One extra transport sweep in the phantom with the fictitious quadrature set using the converged flux moments from *Step 1* to evaluate the scattering source:

$$S_s = \sum_{g=1}^G \sum_{l=0}^L (2l+1) S_{s,g \rightarrow g,l} \left\{ P_l(m_n^{(fic)}) \cdot f_{g,l}^{(con)} + 2 \sum_{k=1}^l \frac{(l-k)!}{(l+k)!} P_l^k(m_n^{(fic)}) \cdot \left[j_{C,g,l}^{k,(con)} \cdot \cos(kj_n^{(fic)}) + j_{S,g,l}^{k,(con)} \cdot \sin(kj_n^{(fic)}) \right] \right\}$$

4. Simulation of the projection image with the fictitious quadrature set using the simplified ray-tracing formulation outside of the phantom
- *Step 1* is completed once and *Steps 2-4* are then repeated for each projection angle desired.

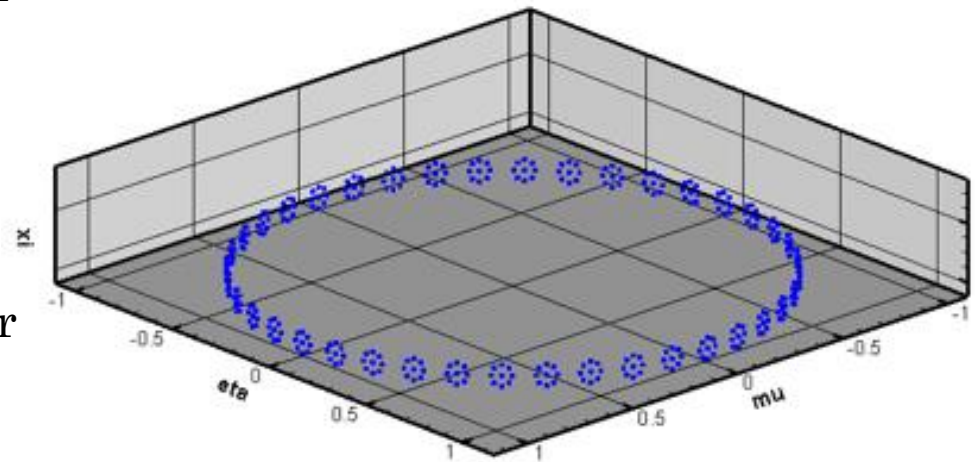
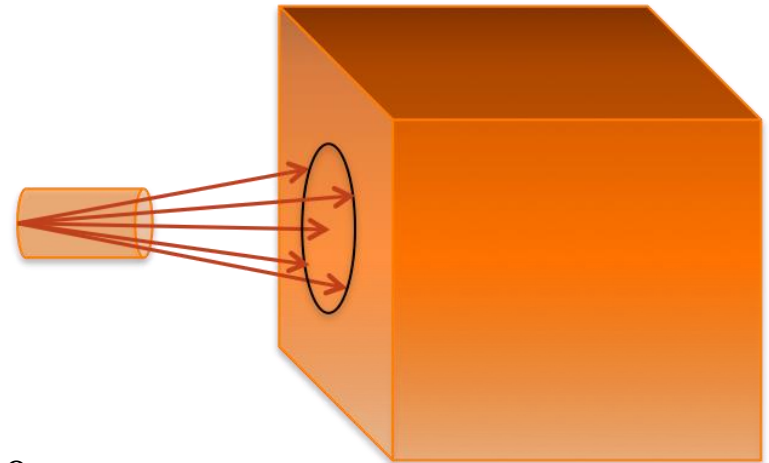


THE TITAN SPECT FORMULATION



○ Circular Ordinate Splitting (COS)

- TITAN feature to approximate the collimator
- Represents an acceptance angle θ about the detector normal within which incoming photons reach the detector
- Split directions made on a circle (or concentric circles) centered on the original projection direction
- Backward ray-tracing from collimator to phantom surface
- Average over original and split directions to approximate collimator blur

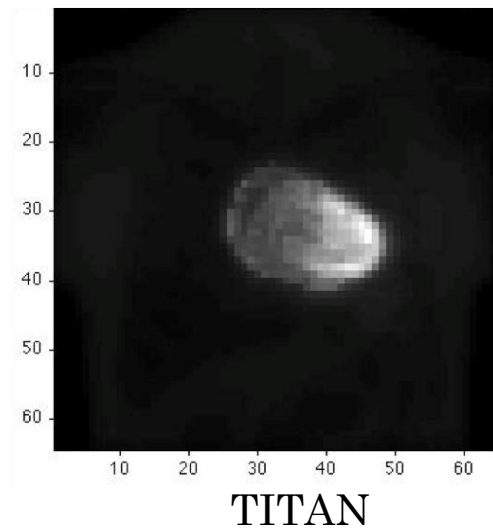
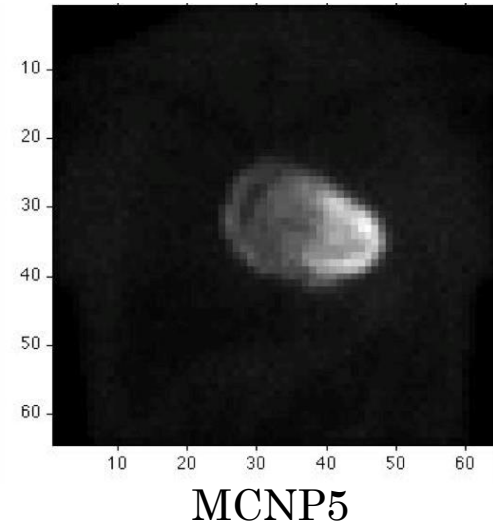


PREVIOUSLY COMPLETED WORK: COMPARING TITAN RESULTS FOR DIFFERENT COLLIMATORS

- TITAN comparison with MCNP5
 - using the NCAT voxel phantom
 - considering different collimator acceptance angles

Acceptance Angle	Maximum relative difference
2.97°	21.3%
1.42°	11.9%
0.98°	8.3%

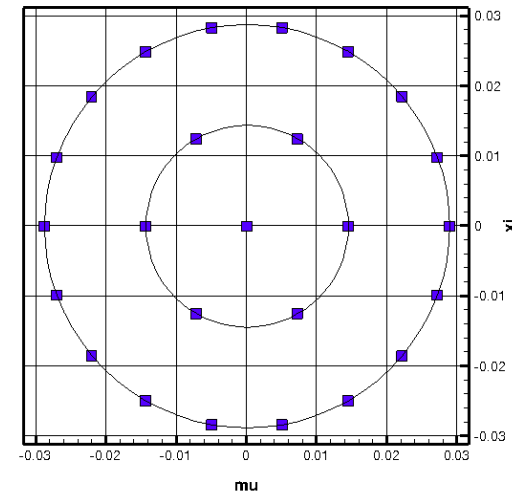
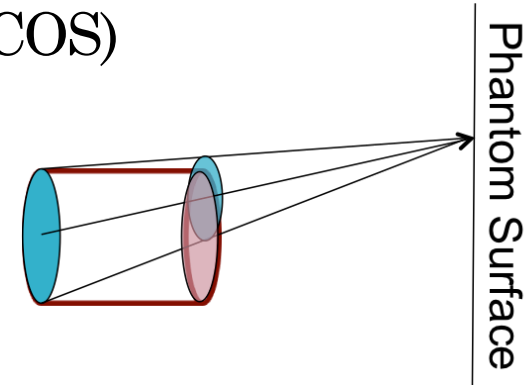
*All MCNP5 data had 1- σ uncertainty $\leq 3\%$ in the heart



THE WCOS ALGORITHM

○ Weighted Circular Ordinate Splitting (WCOS)

- Developed to improve upon the COS collimator representation (especially, for collimators with small aspect ratios)
- Split directions used to calculate a geometry-based weighted average
 1. Project detector surface area to front of collimator
 2. Weight angular flux at phantom surface by overlapping area
- Number of split directions in concentric circles scaled to area
- User specifies collimator parameters (determines radius of outermost circle) and splitting order (i.e., number of directions on innermost circle)

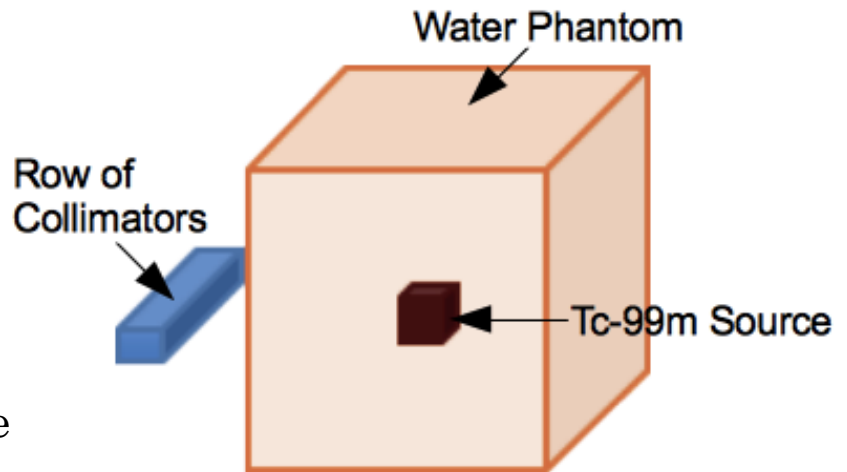


Example: splitting order of 6 with 2 circles



APPLICATION

- Cube of water with a cube of Tc-99m (140.5 keV) at the center
- Cross sections generated using the CEPXS code (20% energy window)
- Multigroup MCNP5 utilizing the CEPXS cross sections
- Model a Low Energy General Purpose (LEGP) collimator & a Low Energy High Sensitivity (LEHS) collimator



Collimator	Acceptance Angle	Detector Pixel Size	Aspect Ratio
LEGP	1.65°	0.210 cm	17.4:1
LEHS	4.29°	0.340 cm	6.7:1



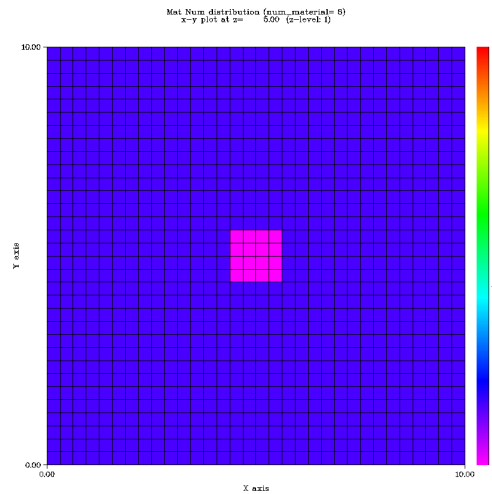
RESULTS

1. Phantom Modeling – Mesh and Quadrature Studies
2. Comparison with Monte Carlo
3. Computation Time & Parallel Performance



RESULTS: PHANTOM MODELING – MESH & QUADRATURE STUDIES

Mesh	Number of Meshes	Mesh Size
Coarse	16x16x16	0.6250 cm
Base	32x32x32	0.3125 cm
Fine	64x64x64	0.15625 cm



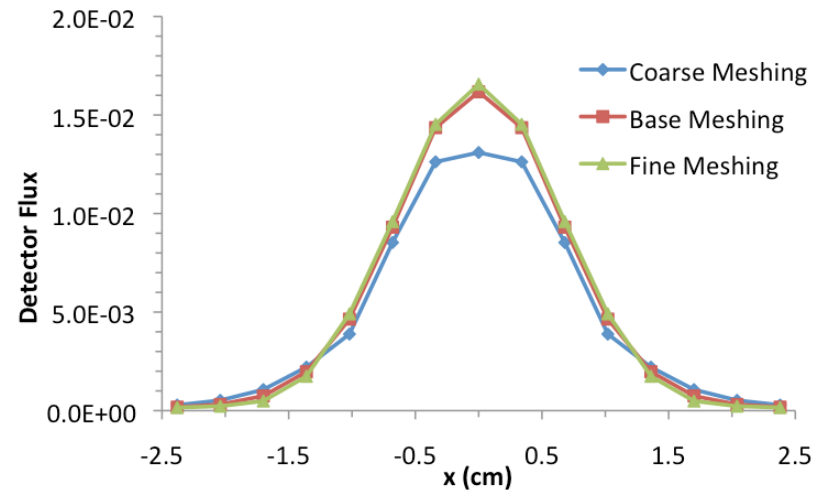
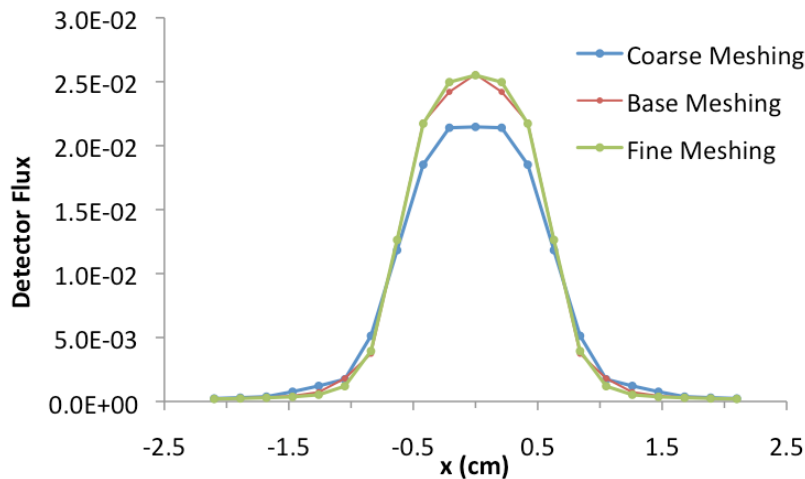
Base Meshing



RESULTS: PHANTOM MODELING – MESH & QUADRATURE STUDIES

LEGP collimator (1.65°)

LEHS collimator (4.29°)



Choose the fine meshing for LEGP and the base meshing for LEHS



RESULTS: PHANTOM MODELING – MESH & QUADRATURE STUDIES

Difference in TITAN detector flux for LEHS collimator

Quadrature Order	Difference Relative to S_{60}	
	Average	Maximum
S_6 (48 directions)	-9.28%	-35.46%
S_{12} (168 directions)	-1.85%	-6.54%
S_{20} (440 directions)	-0.26%	-1.33%
S_{40} (1680 directions)	-0.02%	-0.06%

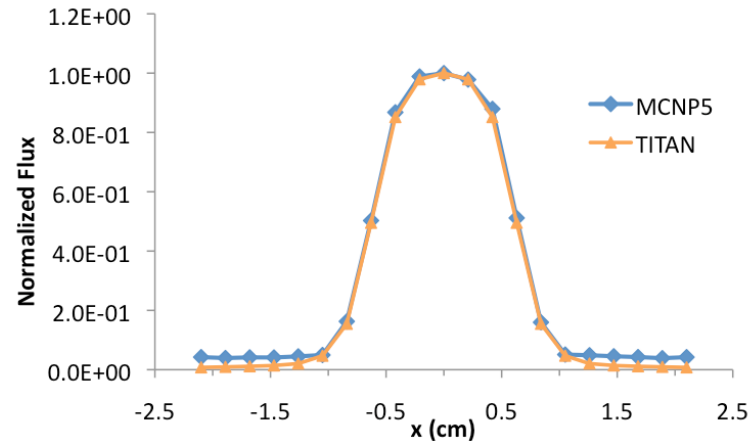
- Same behavior observed for LEGP collimator
- S_{20} level-symmetric quadrature used in all following results



RESULTS: COMPARISON WITH MONTE CARLO

○ LEGP Collimator (1.65°)

- MCNP5 1σ uncertainties 0.8-3.6%
- For normalized fluxes >0.1 , average difference of 2.3%
- No significant difference between original COS and WCOS techniques



FWHM*		Difference
MCNP5	TITAN	
1.27 cm	1.25 cm	-1.2%

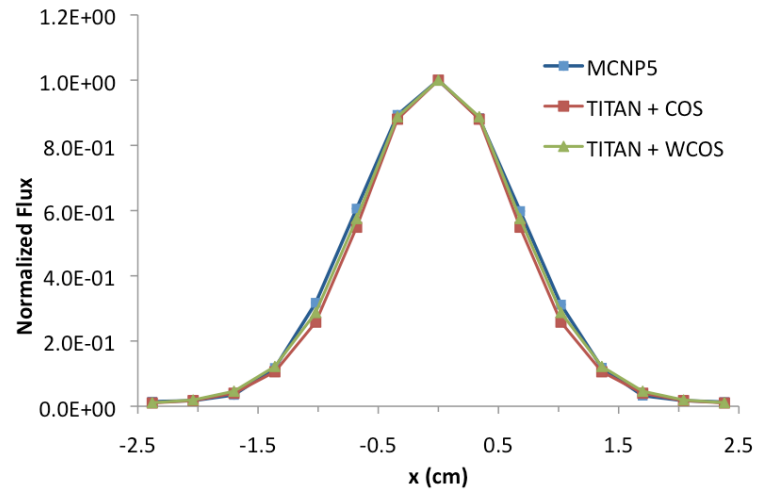
*Full Width at Half Maximum



RESULTS: COMPARISON WITH MONTE CARLO

○ LEHS Collimator (4.29°)

- MCNP5 1 σ uncertainties 0.4-4.4%
- The WCOS technique improves the TITAN solution



Code	FWHM (Relative Difference)	Average Relative Difference
MCNP5	1.60 cm	-
TITAN + COS	1.47 cm (-7.9%)	8.5%
TITAN + WCOS	1.54 cm (-4.0%)	3.8%



RESULTS: COMPUTATION TIME & PARALLEL PERFORMANCE

- All studies were completed on a dedicated PC-cluster:
 - Intel Xeon 2.4 GHz processors
 - 3 compute nodes with 8 processors cores per node
 - 64 GB per node (8 GB per core)
 - 10 Gb network
- Detector dimensions chosen to cover model:
 - LEGP collimator – 42 by 42 detector array
 - LEHS collimator – 30 by 30 detector array
- Parallel Performance Metrics:

Parallel Speedup	$S_p = \frac{\text{Serial Computation Time}}{\text{Parallel Computation Time}}$
Parallel Efficiency	$E_p = S_p / P$
Parallelizable Fraction	$f_p = \frac{P(1 - S_p)}{S_p(1 - P)}$

P = number of processor cores



RESULTS: TITAN-WCOS COMPUTATION TIME & PARALLEL PERFORMANCE

Computation times for LEGP Collimator Case with increasing number of projection images:

Serial Computation Times

Projections	S_N Time (s)	Projection Time (s)	Total Time (s)
4	435	25	455
45	-	298	729
90	-	557	985

Parallel Computation Times on 8 Processor Cores

Projections	S_N Time (s)	Projection Time (s)	Total Time (s)	Parallel Speedup
4	56	6	65	7.0
45	-	38	94	7.7
90	-	76	132	7.5



RESULTS: TITAN-WCOS COMPUTATION TIME & PARALLEL PERFORMANCE

	Number of Processor Cores	Parallel Speedup	Parallel Efficiency	Parallelizable Fraction
1 Node	2	1.95	0.98	0.97
	4	3.87	0.97	0.99
	8	7.47	0.93	0.99
2 Nodes	12	10.97	0.91	0.99
	16	12.44	0.78	0.98
3 Nodes	24	17.07	0.71	0.98

Results for 90 projection angles



RESULTS: COMPARISON OF TITAN WITH MCNP5 COMPUTATION TIME

Computation Times on 8 Processors

Collimator	MCNP5*		TITAN†
	Maximum Uncertainty (1σ)	Computation Time	Computation Time
LEGP	15.4%	46.9 hrs	132 sec
LEHS	9.9%	21.4 hrs	14 sec

*Time with source biasing towards a single detector array

†Time to generate 90 projection images



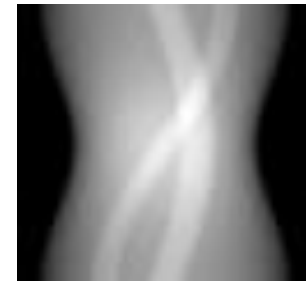
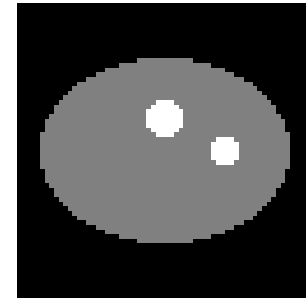
CONCLUSIONS

- The weighted circular ordinate splitting (WCOS) collimator representation has been implemented in the TITAN code
- Algorithm sensitivity to meshing & quadrature order studied
- Solutions benchmarked against MCNP5 for two collimator cases showed excellent agreement
- Parallel behavior was studied and a parallelizable fraction of 98% was found
- Computation times were shown to be on the order of minutes for TITAN and hours/days for MCNP5



ONGOING & FUTURE WORK

- An iterative reconstruction algorithm is being developed to utilize TITAN to model attenuation and scatter in the patient during the forward projection step
- Currently testing reconstruction of a 2-dimensional phantom using TITAN



ACKNOWLEDGMENT

The authors would like to thank Dr. Ce Yi of Georgia Tech for his assistance with the TITAN code



THANK YOU FOR YOUR ATTENTION!



Questions?